On Vagueness, 4D and Diachronic Universalism

Yuri Balashov

Department of Philosophy, University of Georgia, Athens, GA 30602, USA
yuri@uga.edu

[February 28, 2005]

Abstract

I offer a new criticism of the argument from vagueness to four-dimensionalism [Sider 2001]. The argument is modeled after an older argument for mereological universalism [Lewis 1986] and may be looked upon as a tightened-up and extended version of the latter. While I agree with other critics [Koslicki 2003; Markosian 2004] that the argument from vagueness fails precisely because of this affinity, my recipe for dealing with it is different. I reject the assumption, shared by Sider with his opponents, that synchronic composition and ‘minimal diachronic fusion’ are sufficiently similar to use considerations inspired by the analysis of the former to bear on the latter. My objection to a crucial premise of the argument from vagueness turns on the relevant aspect of dissimilarity between these two cases.

I. Introduction

The argument from vagueness to four-dimensionalism (4D), hereafter referred to as the Argument, plays a central role in a recent book-length defense of 4D [Sider 2001: §4.9] (see also Heller [1990: 49ff]). The Argument recently came under fire [Koslicki 2003; Markosian 2004] (see also Hudson [2001: §3.7]). Both the author and the critics acknowledge the paramount importance of the Argument in the case for 4D and against three-dimensionalism (3D). Theodore Sider admits that it is ‘one of the most powerful’ [2001: 120] and ranks it next only to the argument from ‘coincident entities’. Ned Markosian takes it be ‘the most important and powerful argument in the book’ [2004: 665]. And Kathrin Koslicki claims that ‘in the end … everything turns on the [Argument]…. If it were not for [it], there would be a relative stand-off between [3D and 4D], given the rest of Sider’s evidence [2003: 108].’ I suppose many 4Dists and some 3Dists would

1
Yuri Balashov

resist the claim that everything turns on the Argument. But clearly the Argument carries a lot of weight in the persistence debate, as do other considerations related to vagueness (cf. Hawley [2001: chapter 4]).

Below I add my voice to the critics. I hasten to note that although I share with them the belief that the flaws of the Argument can eventually be traced to its kinship with an older argument for mereological universalism [Lewis 1986], my recipe for dealing with the thesis of universalism and my resulting critical strategy are different. Sider’s Argument includes three steps. Step 1 defends a version of David Lewis’s argument [1986: 212f] for unrestricted composition at a time, or synchronic universalism:

(SU) Any class of objects existing at t has a fusion at t.

Step 2 employs considerations parallel to those at play in Step 1 to argue for diachronic universalism—roughly, the view that, for any interval of time (which need not be continuous) and objects existing at various moments in it, there is something they compose over the interval:

(DU) If for all \( t \in T \) there is an object \( x(t) \) existing at \( t \), then there is an object \( y \) existing at all \( t \in T \), but at no \( t \notin T \), such that, at \( t \), \( y = x(t) \).\(^1\)

Finally, Step 3 uses this result to show that temporal parts exist.

Other critics have treated the first two steps as a package deal and then rejected the whole deal by noting that composition may be brutal (Markosian) or that Sider’s support of the important premise that composition is never vague is dialectically weak (Koslicki). In difference from them, I have nothing to say about composition at a time. But I reject the assumption, shared by Sider with his critics, that synchronic composition and what Sider calls ‘minimal diachronic fusion’ are sufficiently similar to use considerations inspired by Step 1 (i.e., the argument for (SU)) to bear on the latter. My objection to a crucial premise of Step 2 turns on the relevant aspect of dissimilarity between these two cases.

If I am right then (a) the Argument is unsound and (b) rejecting (DU) is sufficiently motivated. While the 3Dist may welcome (a) as undermining the general case for 4D, I am inclined to consider both (a) and (b) as encouraging important domestic restructuring in the 4Dist camp. Although most 4Dist are, in fact, diachronic universalists, they need not and, I urge, should not be. If (DU)

\(^1\) As stated, (DU) presupposes (SU). The notion of existing at a time and the relations of parthood and identity at a time are intended to be neutral between 3D and 4D, to avoid prejudging any important issues. For more details, see Sider [2001: §§3.2–3.3 and 4.9.2] and sections II and III below.
On Vagueness, 4D and Diachronic Universalism

proves unhelpful in supporting 4D, it becomes a burden. And shedding the
commitment to diachronic trout-turkeys should, I think, be perceived as a relief,
even if this, by itself, does not help with their notorious synchronic ilk.

I begin by reviewing the Argument and other criticisms of it. I offer my
own critique in section III and discuss some objections and implications in
sections IV and V.

II. The Argument from Vagueness

Step 1 of the Argument, which is a tightened-up version of Lewis’s earlier attack
on restricted composition, employs some technical notions. A case of
composition is a possible situation involving a class of objects at a time, for which
the question of whether they compose anything, or have a fusion, can be raised. A
continuous series of cases is a finite series of cases of composition in which
any two adjacent cases are extremely similar in all respects that may be deemed to
be relevant to composition. A sharp cutoff in a continuous series is a pair of
adjacent cases differing in whether composition occurs.

The premises of Step 1 are as follows:

P1: If not every class has a fusion, then there must be a pair of cases
connected by a continuous series such that in one, composition
occurs, but in the other, composition does not occur.

P2: In no continuous series is there a sharp cutoff in whether
composition occurs.

P3: In any case of composition, either composition definitely occurs,
or composition definitely does not occur.

Suppose composition (at a time) is restricted. Then there is a continuous series
connecting a case of composition to a case of non-composition (P1). By P3, there
must be a sharp cutoff in this series, which is prohibited by P2. Therefore
composition is unrestricted.

P1 is the least problematic of all the premises of Step 1. Sider’s defense of
P2 exploits the intuition that any sharp cutoff would be metaphysically arbitrary.
This can be resisted by someone advocating the brutality of compositional facts
[Markosian 1998; 2004]. Sider’s defense of P3 proceeds by showing that
indeterminacy in composition would result in count indeterminacy in finite worlds—the indeterminacy in the truth-value of a purely logical sentence stating that there are exactly \( n \) objects. Since logical vocabulary, specifically the

\[ ^2 \text{My explication of these notions and the outline of the Argument closely follow Sider}\]
\[ 2001: 122–39. \]
quantifiers, is not vague (the assumption questioned by Koslicki [2003]; but see Sider [2003]), P3 is thereby vindicated.

Step 2 of the Argument is modeled after Step 1 but requires more machinery to state. An assignment \( f \) is a function from times to non-empty classes of objects existing at those times. A \textit{diachronic fusion} of assignment \( f \) (a \textit{D-fusion} of \( f \)) is an object \( x \) that is a fusion-at-\( t \) of \( f(t) \) for every \( t \) in \( f \)'s domain. A \textit{minimal D-fusion} of \( f \) is a D-fusion of \( f \) that exists only at times in \( f \)'s domain.\(^3\,4\)

The premises of Step 2 closely parallel those of Step 1:

\begin{align*}
P1' & : \text{If not every assignment has a minimal D-fusion, then there must be a pair of cases connected by a continuous series such that in one, minimal D-fusion occurs, but in the other, minimal D-fusion does not occur.} \\
P2' & : \text{In no continuous series is there a sharp cutoff in whether minimal D-fusion occurs.} \\
P3' & : \text{In any case of minimal D-fusion, either minimal D-fusion definitely occurs, or minimal D-fusion definitely does not occur.}
\end{align*}

\( P1'\), \( P2'\), and \( P3' \) imply:

\begin{align*}
(U) & : \text{Every assignment has a minimal D-fusion.}
\end{align*}

For suppose they do not. Then there is a continuous series connecting a case where minimal D-fusion occurs to a case where minimal D-fusion does not occur \((P1')\). By \( P3' \), there must be a sharp cutoff in this series, which is prohibited by \( P2' \).

Given \((U)\), Step 3 shows that every object has temporal parts and, hence, that 4D is true. Consider the assignment \( f^* = \langle t, \{x\} \rangle \), where \( x \) is an arbitrary object and \( t \) a time at which it exists. By \((U)\), \( f^* \) has a minimal D-fusion, \( z \). Using the previously established definition of \textit{temporal part} and the contrapositive of the temporally qualified version of the strong supplementation axiom of extensional mereology,\(^5\) Sider then shows that \( z \) is a temporal part of \( x \) at \( t \).\(^6\)

---

\(^3\) Example: Tibbles is a D-fusion of the assignment having two moments in its domain, \( t_1 \) (pre-accident) and \( t_2 \) (post-accident), and assigning two different but overlapping classes of cells composing it at those times. But Tibbles is not a minimal D-fusion of this assignment because Tibbles exists at times other than \( t_1 \) and \( t_2 \).

\(^4\) Important: these notions are 3D-friendly; they are based on temporally qualified mereology, which the 3Dist must be happy to embrace. Specifically, 4D is \textit{not} built into the definition of minimal D-fusion.

\(^5\) Sider defines an instantaneous temporal part of \( y \) at \( t \) as the object \( x \) which (i) exists only at \( t \), (ii) is part of \( y \) at \( t \), and (iii) overlaps at \( t \) everything that is part of \( y \) at \( t \) [2001: 59]. The use of
III. Refutation of P2’

As noted above, other critics reject Step 2 of the Argument because they also reject Step 1, and for roughly the same reasons. Thus Markosian rejects P2’ because he also rejects P2. He notes, rightly, that ‘a necessary condition for an assignment’s having a minimal D-fusion is that the relevant classes of objects all have fusions at the relevant times’ [2004: 669]. Markosian also indicates that it is not sufficient. How important is this?

I take it to be crucial. The thesis of minimal D-fusion universalism could, in principle, be resisted for two different reasons. (a) One could reject (SU); in the present context that would amount to denying that all values of \( f(t) \) have fusions-at-\( t \). (b) One could reject (DU), by arguing that \( f(t) \) fails to have a minimal D-fusion unless the fusion, \( as \ it \ is \ at \ t_1 \), of \( f(t_1) \) bears an appropriate relation to the fusion, \( as \ it \ is \ at \ t_2 \), of \( f(t_2) \).\footnote{For details, see Sider [2001: 138–9] and [2003: 136].} The strategy adopted by Markosian (and, \textit{mutatis mutandis}, by Koslicki) builds on (a) because he already has a reason (i.e., endorsing brute compositional facts) to reject unrestricted composition \textit{at a time}. Although he mentions (b), he does not offer any independent considerations in its support.

My strategy, on the contrary, is built around (b), and (b) alone. I have nothing to say about (a), which I take to be an advantage. Anyone defending (a) has to deal with a host of difficult issues arising from Peter van Inwagen’s ‘special composition question’. I submit, however, that those are quite orthogonal to the issues arising from (b). I insist, therefore, that the reasons for rejecting (DU) may be different from those that might be invoked to resist (SU) and that getting clear on the difference is important for both parties in the debate about persistence.

Recall (section I above) that (SU) is simply the thesis that every class of objects existing at a particular time has a fusion at that time. 4Dists and 3Dists alike should accept this as a fair rendition of unrestricted composition at a time. (DU) is less univocal. In the 4D framework, it is synonymous with the thesis of unrestricted composition \textit{across} time. In the 3D-friendly environment, on the temporally relative notions of \textit{existence-at-t} and \textit{parthood-at-t} makes this definition 3D-friendly. The mereological principle at work states that, if \( x \) and \( y \) exist at \( t \) and \( x \) is not part of \( y \) at \( t \), then \( x \) has a part at \( t \) that does not overlap \( y \) at \( t \) [ibid.: 58].

\footnote{For details, see Sider [2001: 138–9] and [2003: 136].}

\footnote{For the 3Dist, the fusion, \textit{as it is at} \( t_n \), of \( f(t_n) \) is just the fusion-at-\( t_n \) of \( f(t_n) \). For the 4Dist, there will not, in general, be \textit{the} fusion-at-\( t_n \) of \( f(t_n) \) (I thank Ted Sider for clarifying this point). But there will be \textit{the} momentary object (the fusion, \textit{as it is at} \( t_n \), of \( f(t_n) \)), which is the fusion of the class \( f(t_n) \) of momentary objects existing at \( t_n \).}
other hand, it amounts to Sider’s universalism about minimal D-fusions. Both incarnations of (DU) can be resisted on the same grounds—the grounds that are different from, and independent of, any considerations having to do with synchronic composition. My chief concern here is to challenge universalism about minimal D-fusions. I turn, accordingly, to what I take to be the problematic premise of Step 2 of the Argument:

P2': In no continuous series is there a sharp cutoff in whether minimal D-fusion occurs.

What would it take to refute it? Short of brutalism (which I unequivocally condemn), one has to produce an example of a continuous series of cases of minimal D-fusion featuring a motivated sharp cutoff. To be sure, no attempt at motivating it will find much sympathy with the universalist. But that is irrelevant, for converting the universalist is not on the agenda. What is on the agenda is providing a good rationale for anyone inclined to take issue with a crucial step in an argument aimed at establishing (DU) in the first place.

I will develop my example (referred to below as the Example) ‘backwards’, starting with a sharp cutoff and then describing a ‘continuous series’ of cases containing it. The sharp cutoff features a pair of adjacent cases of minimal D-fusion differing in whether minimal D-fusion occurs:

Case 1: \( f_1 = \{(t, \{a\}), t \in T\} \), where \( T \) is \( a \)'s total lifetime

For simplicity, take \( a \) to be an isolated mereological atom. (But nothing turns on its being an atom; see below.) \( T \) is a continuous interval that may be finite or infinite. In Case 1, minimal D-fusion takes place. Indeed, the minimal D-fusion of \( f_1 \) is \( a \) itself throughout its entire life career.

Case 2: \( f_2 = \{(t, \{a\}), t \in T'\} \), where \( T' = T - \{t^*\} \)

In Case 2, minimal D-fusion fails to take place. Motivation: the existence of the object, which would be the minimal D-fusion of \( f_2 \), would violate a fundamental law of physics—the classical law of conservation of matter.\(^8\) Such a fusion would feature an object \( a \) going out of existence at \( t^* \) and popping back into existence ex nihilo.\(^9\) Relatedly, some later phases of such an object would not be connected by immanent causation to its earlier phases.

---

\(^8\) In its local form expressed by the continuity equation \( \frac{\partial \rho}{\partial t} + \nabla (\rho u) = 0 \), where \( \rho \) is the local density of matter and \( u \) is its local velocity.

\(^9\) If there is the first or last moment of \( a \)'s existence make an additional stipulation that \( t^* \) is distinct from \( T \)'s endpoint(s).
Considerations having to do with violating fundamental laws of nature are unlikely to persuade all metaphysicians. The convinced universalist is free to insist that minimal D-fusions such as that of $f_2$ exist; it’s just that they are not the sort of things that physics is interested in (cf. Hudson [2003]). But such a move will not carry much weight at this point of the dialectic, at which the truth of (DU) has not yet been established. It is the universalist who has to convince the opponent that the opponent’s appeal to firmly established results of science and to the notion of immanent causation is unmotivated. And on the face of it, it is rather well motivated.

Moreover, one could bring the motivation into sharper relief by noting an important dissimilarity between ‘unnatural’ synchronic and diachronic fusions. It may be an open question whether physics (or any other science) can supply a satisfactory criterion for restricting synchronic composition. But in any event, the existence of ‘unnatural’ synchronic fusions cannot be ruled out on any ground of their inconsistency with the laws of nature. In contrast, the Example demonstrates that the alleged existence of certain minimal diachronic fusions is strictly incompatible with some such laws.\(^{10}\)

To sum up, Case 1 and Case 2 are two adjacent cases of D-fusion featuring a motivated sharp cutoff in whether minimal D-fusion occurs. (Notice that these cases are as adjacent, and hence the cutoff is as sharp, as they could get: the difference between Cases 1 and 2 is infinitesimal.) Now on to the ‘continuous series’ of cases $\Sigma$ containing the cutoff in question. Let $\Sigma$ be the ordered pair $\langle$Case 1,Case 2$\rangle$. $\Sigma$ satisfies the conditions for being a ‘continuous series of cases’ because (i) $\Sigma$ is finite, (ii) it is a series of cases of minimal D-fusion, and (iii) any two adjacent cases in $\Sigma$ are extremely similar in all respects relevant to minimal D-fusion.

The existence of $\Sigma$ shows that P2’ is false and, hence, Step 2 of the Argument is unsound.

### IV. Some Objections and Replies

**Objection 1:** Using the unit class $\{a\}$ of a single object trivializes the issue of whether that class has a fusion at a time, for a unit class always has a fusion (cf. Sider [2001: 123, note 46]). As a result, we do not have a continuous series connecting a case of composition with a case of non-composition.

**Reply 1:** But we do have a continuous series connecting a case in which minimal D-fusion occurs with a case in which it does not. Our choice of $f_1$ and $f_2$

\(^{10}\) I thank an anonymous referee for suggesting this point of contrast.
does not trivialize the issue of whether such assignments have minimal diachronic fusions. The Example shows that the nature of such fusions is different from the nature of composition at a time. I say that the former, but not the latter, includes broadly causal relations which figure prominently in the fundamental physical laws.

Reply 2: Nothing of substance turns on the choice of a single object. The Example could be modified to include a class of \( n \) objects existing throughout \( T \) and such that, at any time during \( T \), they compose (at that time) a certain object \( a \). For the same reason, nothing substantial turns on \( a \)’s being a mereological atom.

Objection 2: The Example is weird in that it involves a ‘continuous series of cases’ with only two members. This is not what one expects in a discussion of vagueness! What one normally expects is a ‘sufficiently long’ series of cases with the relevant sort of change occurring somewhere in the ‘middle’.

Reply 1: This expectation comes from the Sorites cases, which may serve as a good model for synchronic composition—because the latter may be alleged to turn on small differences in the multigrade relations among many objects \textit{at a time} (e.g., proximity, contact, chemical bonding, etc.)—but not necessarily for diachronic relations, because, as noted above, those relations have a different nature having to do with immanent causation and restrictions imposed by the laws of nature.

A Sorites series of cases of synchronic composition would begin with a clear case of composition (a compact and well-formed collection of atoms, such as an animal body) and end with an alleged case of non-composition (the collection in question scattered throughout the universe). The universalist would then argue that any sharp cutoff ‘somewhere in the middle’ would be metaphysically arbitrary and, hence, all synchronic fusions should be allowed, no matter how scattered and gerrymandered. She could add that the existence of such fusions would not violate any physical laws.

Not so in the case of diachronic fusions, where a motivated sharp cutoff can be associated with the \textit{very first} minute deviation from a clear case of D-fusion, such as one that occurs in the transition from Case 1 to Case 2. Deviations of this sort immediately violate the laws of nature. Thus it should not be surprising that the cutoff occurs at the ‘very first step’ and, hence, no long series of cases is needed to illustrate it.
Reply 2: Nothing in the Argument itself requires that a continuous series be ‘sufficiently long’: P2’ universally quantifies over all continuous series.11

Reply 3: One is free to turn Σ into a ‘sufficiently long series’ by adding more cases of the same sort (i.e., by excluding from T more and more instants). The cutoff would still occur at the very first step. But this should be recognized as a natural feature of a physically motivated restriction on diachronic fusions. The upshot is that Σ is a legitimate series and, thus, a counterexample to premise P2’ of the Argument.

Objection 3: Examples such as the Example could be excluded by imposing topological constraints on the domain of f, say, by restricting such domains to continuous intervals or sums of continuous intervals of time (cf. Sider [2001: 136]). Restrictions of this sort would affect the Argument, but the Argument thus affected ‘would still establish a restricted version of [4D] according to which there exist continuous temporal intervals of arbitrarily small duration. For most [4Dists] that would be [4D] enough [ibid.]’

Reply 1: (a) I do not see immediately how the Argument thus modified would still establish a version of 4D; and Sider does not elaborate. (b) Even if it would, I believe the Example could be modified in a similar way.

Reply 2: But we need not quarrel, for invoking topological restrictions on intervals of time is, in the present context, a red herring. The Example is based on violating a physical law, and not any topological property of time. To set these two issues apart, let us modify Case 2 as follows:

Case 2’: \( f_2’ = \{ \langle t, \{a\} \rangle, t \in T’ \} \cup \{ \langle t*, \{b\} \rangle \}, \)

where (i) \( T’ = T - \{ t* \} \), (ii) \( b \neq a \), (iii) \( b \) has the same intrinsic physical properties as \( a \), (iv) \( b \) is located as close as physically possible to \( a \).

11 Incidentally, the assignments \( f_1 \) and \( f_2 \), responsible for the allegedly ‘weird’ character of the Example because they incorporate a single object and a single moment of time eliminated from \( T \), are not unlike the assignment \( f^* \) in the (elegant and indispensable) Step 3 of the Argument, which also incorporates a single object and a single moment of time. Tit for tat?

It should also be noted that far from all cases of the more traditional vagueness of qualitative predicates invoke, or even evoke, ‘sufficiently long’ Sorites series. The following questions illustrate the ‘non-Sorites’ kind of vagueness: ‘Are prisoners of war residents of an alien country?’ ‘Are skis vehicles?’ ‘Does a glass house have any windows?’ ‘Can you dial on a push button phone’ [Sorensen 2001: 23f].
In Case 2′, minimal D-fusion fails to take place, for the existence of the object, which would be the minimal D-fusion of \( f_2′ \), would violate the laws of motion. Such a fusion (‘aba’, to give it a name) would feature an object ‘jerking’ for an instant from its lawful trajectory. Relatedly, some later phases of aba would not be connected by immanent causation to some of its earlier phases. Cases 1 and 2′ are truly adjacent, even if not infinitesimally close. Consequently, just like \( \Sigma \), the series \( \Sigma′ = \langle \text{Case 1}, \text{Case 2}′ \rangle \) exemplifies a motivated sharp cutoff in whether D-fusion occurs. But the domain of \( f_2′ \) is now a continuous interval of time.

V. Implications

Despite the failure of the Argument, the overall case for 4D, I contend, remains strong. 4D’s ability to provide the best unified solution to the paradoxes of coincidence may still count as the strongest reason in its favor (see, in this connection Sider [2001: chapter 5]; Hawley [2001, chapter 5]; Heller [2000]), and the cumulative weight of other arguments for 4D (for an extensive list of them, see Sider [2001, chapter 4]) is significant.

If the Example achieves its goal, one important implication of it is that defending some version of restricted synchronic composition is no longer a prerequisite for resisting diachronic universalism. Accordingly, there is no pressure for the 4Dist to be a universalist across the board. She is free to banish, not merely disregard, loose collections of unrelated temporal parts (such as certain parts of aba) even if she is not yet prepared to do away with loose collections of spatial parts. While some may view such partial rejection of universalism as arbitrary, I submit that it simply follows the joints of nature. Broadly causal relations linking the state of a particle (or of its temporal part) at one time to its state (or to the state of its other temporal part) at a later time are physically determinate, even if they are not observationally perspicuous and even if they are normally enmeshed with their more problematic synchronic cousins—for example, when we wonder whether a fusion of a certain class of particles at \( t_1 \) is the same object as the fusion of a different such class at \( t_2 \). Particle-by-particle worms can still be disentangled from the mess, and the question of which composite objects are also diachronically related is a separate issue, which eventually boils down to the question of which composite objects comprise which collections of particles at particular moments of time. And that would call for
engagement—both extensive and orthogonal to the agenda of this paper—with the problem of *synchronic* composition.12

References


12 I am indebted to Theodore Sider and the referees for many stimulating comments and helpful suggestions. I also thank James Simmons for useful discussion of the issues raised in this paper. Work on this paper was supported by a senior faculty grant from the University of Georgia Research Foundation.